Prediction of Gear Tooth Crack Propagation Path Based on Pseudo Evolutionary Structural Optimization

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ABSTRACT

In an attempt to reduce the computational requirements on gear crack path prediction an efficient alternative method based on pseudo evolutionary structural optimization (ESO) is proposed in this paper. The novel method is self-evolutionary and does not require prior estimation of stress intensity factors neither initial crack location. During the evolutionary process, instead of removing materials with minimum stress in the design domain as in ESO, elements with maximum tensile stress are progressively eliminated and consequently a crack path is defined. The two-dimensional static analysis involves four finite element models of three successive teeth of a gear section with different backup ratios. The results have shown that the proposed method successfully predicts crack growth direction, which is into the gear rim for backup ratio less than unity or through the tooth foot for back up ratio equal to or greater than unity. The simulated results agree remarkably well with solutions, experimentally and analytically, proposed by previous rigorous procedures.

Keywords: Crack Path Prediction, Tensile Stress, Pseudo Evolutionary Structural Optimization, Finite Element Analysis.

1. Introduction

Gearing systems are known to be the most demanding part in aircraft and aerospace machineries. Due to their severe operating conditions gears are being continually designed in order to provide more efficient and lightweight solutions\textsuperscript{[1]}. To meet these goals manufacturers are constrained to develop gear teeth with sufficient strength, to ensure high wear resistance, and to achieve thin gear rim. However, rim with inadequate thickness often results in gear tooth fatigue due to bending. In gearing, the most frequent fatigue breakdown is caused by tooth bending fatigue\textsuperscript{[2]}. Indeed, as cantilever beam is weakest at its base gear-tooth fractures commonly initiate in the root fillet\textsuperscript{[1]}. After initiation, a crack expands either towards the tooth foot or inside the rim. In the latter case it would disastrous for an operating aircraft because it occasions the disengagement of a propeller or a rotor\textsuperscript{[3]}. Therefore, to prevent eventual casualties, study of crack propagation in gear design has roused a lot of interest.

In the past few years a number of investigations were undertaken to predict gear crack path. Several techniques have been proposed and they are mainly based on Linear Elastic Fracture Mechanics (LEFM)\textsuperscript{[4,6]}. In the aim to investigate crack propagation path on spur gear S. Zouari et al.\textsuperscript{[7]} developed a FORTRAN code associated with a finite element (FE) program and LEFM method. The code reproduced spontaneously a gear section with three successive teeth including a fine meshing and a crack at a tooth foot. However, extensive computations were necessary in order to evaluate the stress intensity factor essential for predicting crack growth. Lewicki and Ballarini\textsuperscript{[3]} performed analytical study of gear rim thickness and gear torque effects on both crack propagation angle and crack direction. They conducted experimental investigation in a gear test rig to support their proposed method. The coupling of LEFM, FE competing based on fracture analysis code (FRANC) with prior assumption of an initial crack on the gear tooth foot allowed calculating mode one (I) stress intensity factors and simulating crack propagation direction. Moreover, to study crack extension Flasker and Pehan\textsuperscript{[8]} carried out FE method of a gearbox and correlated results with experiments. Numerically crack prediction was possible by applying fracture mechanics and estimating the stress intensity factors, which were calculated using virtual crack extension method. Good agreement was found in terms of direction of crack growth. Crack propagation in multi-dimensional structures was also predicted by A.C. Neves et al.\textsuperscript{[9]}. A cracked geometry was initially approximated on the models and stress intensity factors were determined from boundary element method. The accuracy of the crack growth prediction relied upon the exactitude of the stress intensity factors. In order to evaluate the latter stress intensity factors, Blarasin et al.\textsuperscript{[10]} developed FE and Weight function techniques, respectively. Pandya and Parey\textsuperscript{[11]} established a 2-D finite element method with principle of linear elastic fracture mechanics to investigate the crack propagation path for gear pairs with different contact ratio. And then they\textsuperscript{[12]} also simulated the crack propagation in spur gear tooth with different gear parameters, where a linear elastic fracture mechanics based two dimensional FRANC finite element computer program was adopted to simulate the crack propagation. In these models, two and three dimensional FE
models of gear teeth cracked specimens were considered. Both methods displayed similarity with experimental results of crack propagation.

Accurate simulation on the tooth root crack propagation path could help capturing the right dynamic vibration features of the geared system. There has been numerous literatures concerning the vibration responses of gear systems with tooth root crack based on the knowledge of crack propagation path shape. Chaari et al. [13] studied the dynamic responses of a planetary gear with tooth pitting and crack failures by changing the amplitude and phase of the mesh stiffness simplified as rectangular waves. Wu et al. [14] established an analytical model for calculation of spur gear mesh stiffness based on the potential energy theory, and employ this model to investigate the dynamic responses of the gear system with different crack level. However, the former crack models were usually concentrated on its propagation along crack depth while the propagation along tooth width was ignored. Chen and Shao [15, 16] developed a new analytical mesh stiffness calculation model which enables the simulation of the crack propagating along both crack depth and tooth width. This work has extended the analytical calculation models for cracked gear mesh stiffness from 2-D to 3-D. And the authors [17] also derived the mesh stiffness calculation formulas for internal gear pairs based on the potential energy principle where a straight line propagation path was assumed. Ma et al. [18] evaluated the mesh stiffness of a cracked spur gear where different crack propagation paths, namely the straight and parabolic curved, were considered and compared. And further they [19] carried out some investigations on the effect of the tooth root crack on the mesh stiffness and the dynamic vibrations of the gear system, where the cracks propagating through gear tooth and gear rim were simulated by finite element method. Mohammed and Rantatalo [20] employed the mesh stiffness calculation model proposed by Chen and Shao in Ref. [15] to investigate the fault feature of cracked spur gear pairs.

As highlighted in the aforementioned literatures the stress intensity factor is a key parameter in fracture mechanics for predicting crack growth. Evaluated with various techniques, it allows determining the crack geometry such as initiation angle, propagation angle, crack direction, and eventually crack length. However, although there are exciting developments as mentioned above those methods involve tremendous variables and equations, and/or coupling different computer software packages to make crack propagation predictions possible. Therefore, in an attempt to achieve an optimal and less extensive crack assessment on gear tooth we propose a method based on Evolutionary Structural Optimization (ESO) process that we refer here as pseudo ESO. There are three stages within a fatigue failure to be studied closely. They are the origin of the fracture, the progression under successive cycles of loading, and final rupture of the part when the spreading crack has sufficiently weakened the section [2]. The present study focuses on the second stage that is to predict the direction of crack progression. We carry out static analysis of four two-dimensional finite element models of three successive teeth of spur gear section with various backup ratios. Crack propagation paths are defined by progressive elimination of maximum tensile stressed elements of the gear sections. This is at variance with the ESO method where minimum Von Mises stressed materials are removed from a design domain in order to optimize a structure. The proposed method is self-evolutionary and prior estimations of stress intensity factors and initial crack location are unnecessary.

In section 2 we address the technical aspects of the proposed pseudo-ESO method. Simulations are performed in section 3 while section 4 analyzes and discusses the results. Finally, concluding remarks are made in section 5.

2. Proposed pseudo-ESO method

ESO was first introduced in 1993. It is a straightforward evolutionary concept used for shape and layout optimization of structures. Because a full description of ESO process is beyond the scope of this paper, the reader is referred to Refs. [21-23]

Figure 1 depicts a single tooth loading conditions of the initial design domain of the model system. As gear tooth is essentially a stubby cantilever beam, its base exhibits compressive stress on one side and tensile stress on the loaded opposite side. When gear tooth breaks, it usually fails by a crack at the base of the tooth on the tensile-stress side [7]. The critical point in the root fillet of the gear can be found by inscribing a parabola. This is point A in Fig. 1. Indeed, A becomes the most critically stressed location when the gear tooth is loaded at the application point of Fn. Inscribed parabola is tangent to root fillet at point A and has its origin where load vector cuts the center line. The load $F_n$ normal to the tooth surface has a tangential component $F_t$ and a radial component $F_r$.

In most materials, a tensile stress is more damaging than a somewhat higher compressive stress. The component stress $\tau$ can be expressed by the Lewis formula and may be written as [1]:

$$\tau = \frac{P \cdot F}{LY}$$

(1)

where, $F$ stands for applied load, $P_0$ diametrical pitch ($P_0 = \pi / p$) with $p$ circular pitch, $L$ tooth width, $Y$ ($Y = y \pi$) with $y$ the Lewis factor. $y$ can be expressed as [1]:

$$y = \frac{2x}{3p} \quad \text{with} \quad x = \frac{t^2}{4l}$$

(2)

The variables $t$ and $x$ are obtained from construction lines (see Fig. 1).

![Figure 1](image)

Figure 1. Critical stress location. $l$, $t$ and $x$ are obtained from construction lines. Fr and Ft are radial component and tangential component of Fn, respectively [1]

During the evolutionary process of the proposed pseudo-ESO method, instead of eliminating progressively low stressed material in the structure as in ESO, maximum tensile stressed
elements are here removed from the design domain using FE analysis. Adopting the principal stress as rejection criterion (RC) instead of Von Mises stress (commonly used in ESO), the elements with high order tensile stress can be eliminated successively resulting in a crack path definition. The condition of element removal is satisfied by the following equation:

$$\tau_{N,e} \leq RR \times \tau_{N,max}$$

(3)

where, \(\tau_{N,e}\) is the tensile stress or selected criterion of element \(\varepsilon\) and \(\tau_{N,max}\) the maximum tensile stress or selected criterion of the structure. \(RR\) stands for rejection ratio or rate of removed material.

Because the tensile stress is more critical than compressive stress as reported in Ref [1], its effect can result in crack initiation at the tooth root. Hence, the tensile stress is taken as \(RC\) in Eq. (3). In order to avoid big jump during the evolution process i.e. excessive suppression of elements, the initial rejection ratio \((RR_0)\) should be big enough, approximately ninety nine percent (-99%) of the maximum element stress value. Note that this is at variance with the ESO where \(RR_0\) should be less than 1% of the highest element stress value [23] on the structure. An evolutionary rate \(ER\) is also introduced. In contrast to ESO, the \(ER\) is here subtracted to the \(RR\). Therefore, the evolutionary process can be advanced as:

$$RR_{SS+1} = RR_{SS} - ER$$

(4)

where \(RR_e\) is \(RR\) when steady state (SS) is reached. In other words the element removal cycle is rerun with the same \(RR\) value till there are no higher stressed elements to be suppressed.

**Figure 2.** Flow chart of the pseudo-ESO method

The full flow chart diagram of the proposed method is shown in Fig. 2. It depicts the iterative procedure necessary to define the crack path. At the first stage a finite mesh is required as the smallest the mesh size the more accurate are the results. At the fourth stage, if a steady state in not reached i.e. if there is still an element satisfying Eq. (3), the \(RR\) is maintained and the concerned elements are vanished. However, a steady stage condition is fulfilled whenever after material removals an element presents a tensile stress less than the current maximum tensile stress times the rejection ratio. In this latter condition a new \(RR\) has therefore to be set according to Eq. (4), which leads to the next removal step (SS+1). The procedure take place again as its purpose is to predict crack path by element elimination. The cycle stops at the steady state where the crack path matches the critical crack length defined as 86% of the tooth thickness [24].

The effectiveness of the method lays on its simplicity, accuracy and time saving. The design domain does not require to be remeshed. Furthermore, unlike to LEFM method and other techniques reported in literature, the crack initial parameters are here not relevant.

### 3. Simulations

Predictions of spur gear tooth crack propagation in this present paper are correlated with the analytical and experimental studies performed in the NASA Lewis Spur Gear fatigue Rig [3] in where gears with various backup ratios were tested to validate crack path. Therefore, in the present models identical geometrical parameters of gears are employed with same backup ratios defined as rim thickness divided by tooth height. Table I lists the basic gear parameters. Different models are presented with backup ratio of 0.3, 0.5, 1.0 and 3.3, respectively. The FE models built without initial crack, which one example is shown in Fig. 3, are two-dimensional spur gear with three successive teeth. They are made of triangular elements, 8-node and plane stress. The material used is steel. To specify the boundary conditions, all nodes on the two radial lines defining the ends of the segment are selected and given zero displacement in all directions. In order to conduct the static analysis a full static load of \(W_n = 4200N\) was applied on the second tooth profile of the FE model. The application point coincided with highest point of single tooth contact (HPSTC) on the healthy tooth and consequently the load produced the largest bending stress.

**Table I. Test gear geometry**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of teeth</td>
<td>28</td>
</tr>
<tr>
<td>Module (mm)</td>
<td>3.2</td>
</tr>
<tr>
<td>Circular pitch (mm)</td>
<td>9.98</td>
</tr>
<tr>
<td>Whole depth (mm)</td>
<td>7.62</td>
</tr>
<tr>
<td>Addendum (mm)</td>
<td>3.20</td>
</tr>
<tr>
<td>Pressure angle (°)</td>
<td>20</td>
</tr>
<tr>
<td>Pitch diameter (mm)</td>
<td>88.90</td>
</tr>
<tr>
<td>Outside diameter (mm)</td>
<td>95.25</td>
</tr>
<tr>
<td>diameter (mm)</td>
<td>5.49</td>
</tr>
<tr>
<td>Backlash reference (mm)</td>
<td>0.25</td>
</tr>
<tr>
<td>Tooth and rim width (mm)</td>
<td>6.35</td>
</tr>
<tr>
<td>Hub width (mm)</td>
<td>19.05</td>
</tr>
</tbody>
</table>

The proposed method simulations are performed through the ANSYS software package. An example of the natural evolutionary structural procedure is shown in Fig. 4 for a backup ratio equal to 1.0. The stress distribution of the model after using FE analysis is obtained and the maximum tensile stress is localized at the tooth filet [Fig. 4(a)]. By applying the novel algorithm based on structural optimization procedure on the uncracked teeth model (without initial failure) the elements with
maximum tensile stress are deleted progressively to define the crack path. An initial $RR_0$ of 99% is assumed and an $ER$ is of 1% is considered. The following snapshots show the different stages of the evolution and provide a clear view of the evolutionary history of the cracked tooth.

**Figure 3.** Finite element modelling of a three successive teeth

With the chosen applied load, on average three iterations are needed to achieve a steady state for each $RR$ decrement. The tensile stress distribution can be observed in these figures with a maximum stress at the crack tip. As described in Fig. 5 below the crack path follows the highest tensile stress which is always located at the crack tip at each crack length extension. $\tau_{N,S-2}$, $\tau_{N,S-1}$ and $\tau_{N,S}$ are maximum instantaneous tensile stresses at a crack tip corresponding to $C_0$, $C_{S-2}$, $C_{S-1}$ and $C_S$, maximum crack propagation length at each stage $S$ of the computation (Fig. 2), respectively. Note that at $C_0$ a crack has not yet been initiated. The maximum number of steps required for the crack to propagate from the initial size to the critical crack length throughout the tooth root is obtained when the crack reaches a failure threshold: 86% of the tooth thickness [24].

**4. Validation of the pseudo-ESO method**

With regards to backup ratio $m_z$, a crack may grow through the tooth foot or through the rim. Simulated results in Fig. 6 show cracks that propagate through the teeth and not the rims for $m_z = 1.0$, and

**Figure 4.** Evolutionary crack path towards the tooth root

Conversely, cracks will propagate through the rim for $m_z = 0.3$. Correlated experimental and analytical results in Fig. 9 match well with performed simulations in Fig. 8. From Ref. [3] it is reported that at this backup ratio of 0.3 i.e. tooth height greater
than rim thickness; crack propagated through the rim for all initial crack angle orientations. This is consistent with Fig. 8(a). However, discrepancy arises for \( m_b = 0.5 \). Simulation of the present paper and experiment in Ref. [3] produce rim fracture while the analytical method [11] shows crack propagation through the tooth foot. According to Lewicki and Ballarini [11], at this value of backup ratio, the analysis displayed inconsistency (tooth or rim fracture) while the experiments exhibited rim fractures depending on the initial conditions. \( m_b = 0.5 \), therefore, leads to a transition case. Furthermore, tooth load location but also the presence of surface discontinuities, notches, grooves, and subsurface imperfections importantly affected the crack path when rim thickness is equal to half tooth height.

![Figure 8](image1)

**Figure 8.** Crack path simulation towards the rim for: a) \( m_b = 0.3 \), and b) \( m_b = 0.5 \)

![Figure 9](image2)

**Figure 9.** Crack path through the rim [3]: 1 Prediction, 2 Experiment

Crack length has long been evaluated from the Paris-Erdogan law [25,26]. With the pseudo-ESO method the crack size at each steady stage can be obtained considering the removed element sizes. As this study focuses on prediction of crack direction, crack depth investigation is reserved for future work. From the pseudo-ESO simulation results one can note that initial crack which is defined by crack initiation angle and crack length but also the cyclic procedure for crack parameters determination were not necessary to follow crack paths.

**5. Conclusions**

The purpose of this article was to contribute to the study of gear tooth crack path prediction by proposing a new approach termed as pseudo-ESO method. The idea is based on an evolutionary procedure for structural optimization (ESO) and computer simulation has been used for the analysis. Four finite element models of two-dimensional spur gear with three successive teeth and different backup ratio were analyzed. A static load was applied on the tooth profile at the highest point of single tooth contact on the healthy tooth and the stress distribution obtained. Maximum tensile stress materials located at the tooth fillet were chosen as rejection criterion. Progressive elimination of highest stressed elements after each element removal step allows the iterative procedure to progress till the final crack length reaches a failure threshold. To validate the effectiveness and the efficiency of the new based ESO technique, results from the simulation were correlated with published work in literature. Surprisingly, they remarkably agree well with solutions obtained by analytical methods based Linear Elastic Fracture Mechanics and experimental testing, respectively. Simulation results confirmed that if tooth height is less or equal to rim thickness crack propagates through the tooth. However, crack will expands into the rim if tooth height is greater than rim thickness.

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**References**

15. Z. Chen, Y. Shao, Dynamic simulation of spur gear with tooth root crack propagating along tooth width and crack depth, Engineering Failure Analysis, 18(8), 2149–2164 (2011).